Journal of Engineering Physics and Thermophysics, Vol. 70, No. 3, 1997

FRICTION-HEAT DISTRIBUTION IN A ROD-DISK TRIBOSYSTEM

A. A. Evtushenko, O. M. Ukhanskaya, and R. B. Chapovskaya

UDC 539.377

The coefficient of heat distribution in the zone of contact of the end of a cylindrical rod with the face of a rotary disk is determined.

Choice of the Calculation Model. We will consider a tribosystem consisting of a round rod (pin) one end of which rubs against the face of a rapidly rotating steel disk (Fig. 1). In the zone of contact the work against friction forces generates thermal energy and, as a result, a high temperature can develop. Without loss of generality, we assume that:

1) axial heat conduction can be neglected, since the disk thickness is small as compared to the radius. Therefore, the temperature of the disk will depend only on the radial and angular coordinates;

2) the angular speed of the disk is considerably greater than the ratio of the thermal diffusivity of the disk to its radius ($\omega R^2/(2k) > 100$);

3) the intensity of the heat flux is constant in the zone of contact;

4) convective heat transfer proceeds from the face and side surfaces of the disk to the surrounding medium. Heat transfer coefficients are constant and $\mathrm{Bi} \le 1$;

5) in the zone of contact the rod temperature is equal to the mean temperature of the disk surface.

With the assumptions made, we will consider the two-dimensional quasistationary heat conduction problem of a thin disk with a constant-power heat source moving quickly over its face.

Quasistationary Heat Conduction Problem for the Disk. The geometry of the problem and the boundary conditions are shown in Fig. 2. Assume that the polar coordinate system (r, θ) is rigidly connected with the heat source and the disk is rotating with a constant angular speed ω relative to this coordinate system. Taking into consideration assumptions 1) – 3), we built a solution of the heat conduction equation [1]

$$\frac{\partial^2 T^*}{\partial p^2} + \frac{1}{p}\frac{\partial T^*}{\partial p} - \sigma T^* = \mathrm{Pe}\frac{\partial T^*}{\partial \theta}, \tag{1}$$

satisfying the boundary condition

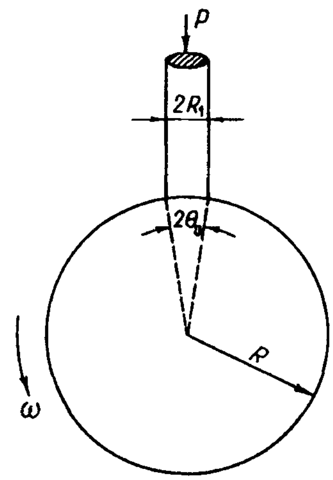
$$\frac{\partial T^*}{\partial p} = \begin{cases} 1, & |\theta| \le \theta_0, \quad p = 1, \\ -\mathrm{Bi}\,T^*, & |\theta| > \theta_0, \quad p = 1. \end{cases} \tag{2}$$

Here

$$\mathrm{Pe} = \frac{\omega R^2}{k}, \quad \mathrm{Bi} = \frac{hR}{K}, \quad \mathrm{Bi}' = \frac{h' R}{K}, \quad p = \frac{r}{R}, \quad \Delta = \frac{\delta}{R}, \quad \sigma = \frac{\mathrm{Bi}'}{\Delta}, \quad T^* = \frac{KT}{qR}. \tag{3}$$

Performing a finite Fourier transformation [2] in Eqs. (1) and (2)

I. Franko State University, L'vov, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 3, pp. 500-505, May-June, 1997. Original article submitted July 19, 1995.



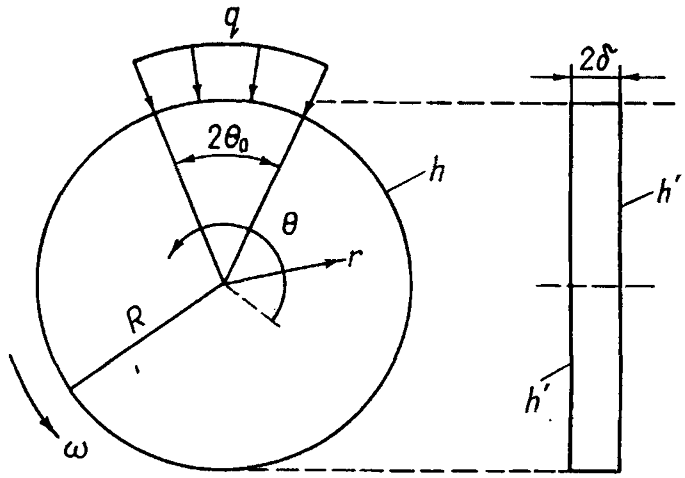


Fig. 1. Geometry of the problem.

Fig. 2. Diagram of rotating cylindrical rod (geometry and thermal boundary conditions).

$$\overline{T}^*(p, n) = \frac{\varepsilon}{2\pi} \int_{-\pi}^{\pi} T^*(p, \theta) \exp(-in\theta)\, d\theta, \quad \varepsilon = \begin{cases} 1, & n = 0, \\ 2, & n \geq 1, \end{cases}$$

we arrive at

$$\frac{\partial^2 \overline{T}^*}{\partial p^2} + \frac{1}{p} \frac{\partial \overline{T}^*}{\partial p} - \lambda^2 \overline{T}^* = 0, \quad \lambda^2 = \sigma + in \,\mathrm{Pe}, \tag{4}$$

$$\left. \frac{\partial \overline{T}^*}{\partial p} \right|_{p=1} = -\mathrm{Bi}\, \overline{T}^* + \frac{\varepsilon}{2\pi} \int_{-\theta_0}^{\theta} [1 + \mathrm{Bi}\, T^*(1, \theta)] \exp(-in\theta)\, d\theta. \tag{5}$$

A solution of Eq. (4) restricted at the disk center and satisfying condition (5) has the form

$$\overline{T}^*(p, n) = \frac{I_0(\lambda p)}{\mathrm{Bi}\, I_0(\lambda) + \lambda I_1(\lambda)} \frac{\varepsilon}{2\pi} \int_{-\theta_0}^{\theta} [1 + \mathrm{Bi}\, T^*(1, \theta)] \exp(-in\theta)\, d\theta.$$

Next, we pass to the inverse transform of the sought solution of problem (1), (2) with the help of the formula

$$T^*(p, \theta) = \mathrm{Re} \sum_{n=0}^{\infty} \overline{T}^*(p, n) \exp(in\theta),$$

which yields

$$T^*(p, \theta) = \frac{\varepsilon}{2\pi} \mathrm{Re} \sum_{n=0}^{\infty} \frac{\int_{-\theta_0}^{\theta} [1 + \mathrm{Bi}\, T^*(1, \theta')] \exp[-in(\theta' - \theta)]\, d\theta'}{\mathrm{Bi} \dfrac{I_0(\lambda)}{I_0(\lambda p)} + \lambda \dfrac{I_1(\lambda)}{I_0(\lambda p)}}. \tag{6}$$

At $\mathrm{Bi} \leq 1$ the term $\mathrm{Bi}T^*$ under the integral in solution (6) can be neglected [3]. Thus, a surface temperature ($p = 1$) of the disk is equal to

$$T^*(1,\theta) = \frac{\varepsilon}{2\pi} \operatorname{Re} \sum_{n=0}^{\infty} \frac{\int\limits_{-\theta_0}^{\theta} \exp\left[-in(\theta' - \theta)\right] d\theta'}{\mathrm{Bi} + \lambda \dfrac{I_1(\lambda)}{I_0(p)}}. \tag{7}$$

At $n = 0$ it follows from equality (7) that

$$T^*(1,\theta) = \frac{\theta_0}{\pi M_0}, \quad M_0 = \mathrm{Bi} + \sqrt{\sigma}\, \frac{I_1(\sqrt{\sigma})}{I_0(\sqrt{\sigma})}. \tag{8}$$

But for $n \geq 1$,

$$T^*(1,\theta) = \frac{1}{\pi} \operatorname{Re} \sum_{n=1}^{\infty} (\mathrm{Bi} + \lambda)^{-1} \int\limits_{-\theta_0}^{\theta} \exp\left[-in(\theta' - \theta)\right] d\theta',$$

where it is taken into consideration that $I_1(\lambda)/I_0(\lambda) \cong 1$ for large $n\mathrm{Pe}$ values [4].

Since

$$\operatorname{Re}\left[(\mathrm{Bi} + \lambda)^{-1} \int\limits_{-\theta_0}^{\theta} \exp\left[-in(\theta' - \theta)\right] d\theta'\right] =$$

$$= \frac{\left[\mathrm{Bi} + D\cos\left(\dfrac{\xi}{2}\right)\right] \cos\left[n(\theta' - \theta)\right] - D \sin\left(\dfrac{\xi}{2}\right) \sin\left[n(\theta' - \theta)\right]}{\mathrm{Bi}^2 + D^2 + 2\mathrm{Bi}\, D\cos\left(\dfrac{\xi}{2}\right)}, \quad n \geq 1,$$

then relation (6) at $n \geq 1$ can be represented in the form

$$T^*(1,\theta) = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{L_n}{nM_n}, \tag{9}$$

$$D = (\sigma^2 + n^2\mathrm{Pe}^2)^{1/4}, \quad \xi = \arctan\left(\frac{n\,\mathrm{Pe}}{\sigma}\right),$$

$$L_n = \left[\mathrm{Bi} + D\cos\left(\frac{\xi}{2}\right)\right] S_n + D\sin\left(\frac{\xi}{2}\right) C_n, \quad M_n = \mathrm{Bi}^2 + D^2 + 2\mathrm{Bi}\, D\cos\left(\frac{\xi}{2}\right),$$

$$S_n = \sin\left[n(\theta_0 - \theta)\right] + \sin\left[n(\theta_0 + \theta)\right], \quad C_n = \cos\left[n(\theta_0 - \theta)\right] - \cos\left[n(\theta' + \theta)\right].$$

With allowance for relations (3), (8), and (9), the temperature distribution in the disk is given by the formula

$$T(R,\theta) = \frac{qR}{K} T^*(1,\theta), \tag{10}$$

$$T^*(1,\theta) = \frac{1}{\pi}\left(\frac{\theta_0}{M_0} + \sum_{n=1}^{\infty} \frac{L_n}{nM_n}\right). \tag{11}$$

Based on the solution of (10), (11), we find the mean temperature in the heating region

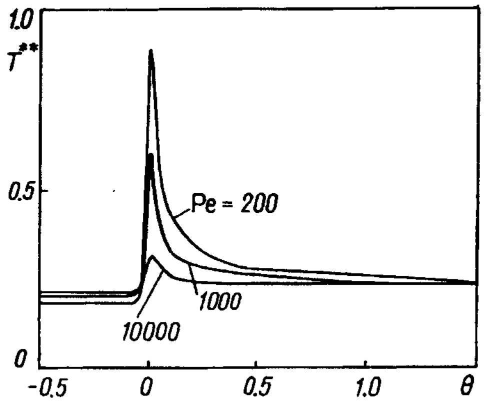


Fig. 3. Distribution of disk surface temperature for $\theta_0 = 0.02$ rad, Bi = 1, Bi$'$ = 0.1, $\Delta = 0.1$ at different Pe numbers.

$$T_a = \frac{1}{2\theta_0} \int_{-\theta_0}^{\theta} T(R, \theta)\, d\theta = \frac{qR}{K} T_a^*, \tag{12}$$

$$T_a^* = \frac{1}{\pi} \left[\frac{\theta_0}{M_0} + \frac{2}{\theta_0} \sum_{n=1}^{\infty} \frac{\left[\mathrm{Bi} + D \cos\left(\frac{\xi}{2}\right)\right] \sin^2 (n\theta_0)}{n^2 M_n} \right]. \tag{13}$$

Stationary Heat Conduction Problem for the Pin. We will construct a solution of the one-dimensional heat conduction equation [1]

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{2h_1}{K_1 R_1} T_1 = 0, \tag{14}$$

satisfying the boundary conditions

$$K_1 \left. \frac{\partial T_1}{\partial x} \right|_{x=0} = -q_1, \tag{15}$$

$$T_1 = 0, \quad x = l. \tag{16}$$

A solution of the boundary-value heat condution problem (14)-(16) is as follows

$$T_1(x) = \frac{q_1 l}{K_1 \varphi} \frac{\mathrm{sh}\, [\nu (l - x)]}{\mathrm{ch}\, (\varphi)},$$

$$\nu = \sqrt{2\mathrm{Bi}_1}/R_1, \quad \mathrm{Bi}_1 = h_1 R_1 / K_1, \quad \varphi = \nu l.$$

Hence the temperature at the end $x = 0$ of the rod is:

$$T_1 = \frac{q_1 l}{K_1 \varphi} \mathrm{th}\, (\varphi). \tag{17}$$

Heat Distribution Coefficient. The amount of heat transferred separately to the rod and to the disk is determined, based on hypothesis 5), by equating the mean temperatures of the disk and the rod in the zone of contact, i.e., $T_a = T_1$. Then from relations (12) and (17) it follows that

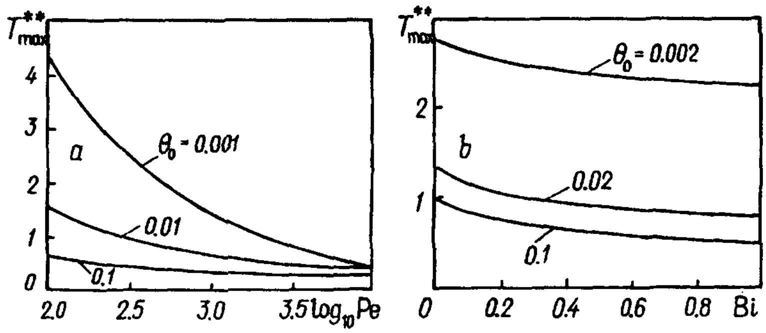


Fig. 4. Maximum surface temperature of disk as a function of the Peclet (Bi = 1, Bi$'$ = 0.1, Δ = 0.1) a) and Biot numbers (Pe = 200, Bi$'$ = 0.1, Δ = 0.1) b) at different angular widths of the heat source.

$$\frac{qR}{K} T^*_a = \frac{q_1 l}{K_1} \frac{\text{th}(\varphi)}{\varphi}, \tag{18}$$

where the dimensionless averaged temperature of the disk surface T^*_a is given by formula (13).

The coefficient of heat distribution between the pin and the disk is determined as

$$\eta = \frac{\gamma}{1+\gamma} = \frac{Q}{Q+Q_1}, \quad 0 \le \eta \le 1, \tag{19}$$

$\gamma = Q/Q_1$, $Q = qA$, $Q_1 = q_1 A_1$. Note that at $\eta = 0$ the entire heat generated in the zone of contact is transferred to the pin, while in the case $\eta = 1$, to the disk. From equality (18) we obtain

$$\gamma = \frac{AKl}{A_1 K_1 R} \frac{\text{th}(\varphi)}{\varphi T^*_a}, \tag{20}$$

Since $R_1 \cong R\theta_0$, the area of the heating zone on the disk surface is equal to $A \cong 4\delta R\theta_0$, and, as a result

$$\frac{A}{A_1} \cong \frac{4\delta R\theta_0}{\pi R_1^2} \cong \frac{4}{\pi} \frac{\Delta}{\theta_0},$$

while from Eq. (20) it follows that

$$\gamma = \frac{0{,}9K\Delta}{K_1 \sqrt{\text{Bi}_1}} \frac{\text{th}(\varphi)}{T^*_a}.$$

Results and Discussion. Figure 3 shows the dimensionless surface temperature of the disk $T^{**}(\theta) = T^*(1, \theta)/\theta_0$ as a function of the angular coordinate for the heat source with an extension of 0.02 rad. for several Pe numbers (3). It is seen that the local temperature rise near the source decreases with increasing Pr. The surface temperature in front of the source is constant, while behind the heating region it, after the rise, gradually decreases approaching its value in front of the source. Its maximum is reached at $\theta = \theta_0$.

It should be noted that the number of terms in series (11) required to attain a prescribed accuracy highly depends on the an angular width of the zone of contact. Narrow heating regions need a larger number of terms. For the indicated value $\theta_0 = 0.02$ rad, from 10^2 to 10^3 terms are required.

The maximum dimensionless temperature of the disk surface $T^{**}_{max} = T^{**}(\theta_0)$ versus Pe numbers for different widths of the contact zone is shown in Fig. 4a. It is seen that at low speeds of rotation, when the heating time of the disk increases, thermal conductivity in radial direction will be high and the temperature in the zone of contact increases.

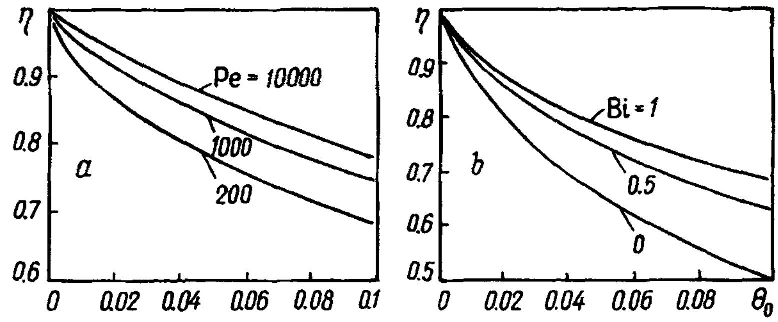


Fig. 5. Heat distribution coefficient versus angular width of the zone of contact at different Peclet a) (Bi = 1, Bi$'$ = 0.1, Δ = 0.1, K/K_1 = 1, Bi$_1$ = 1, l/R_1 = 6) and Biot b) (Pe = 100, Bi$'$ = 0.1, Δ = 0.1, K/K_1 = 1, Bi$_1$ = 1, l/R_1 = 6) numbers.

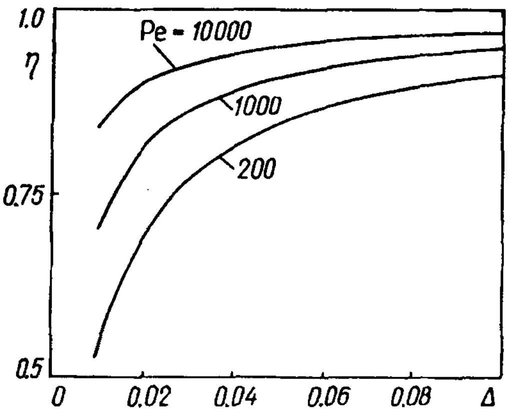


Fig. 6. Heat distribution coefficient as a function of the dimensionless disk half-thickness for Bi = 1, θ_0 = 0.01, Bi$'$ = 0.1, Δ = 0.1, K/K_1 = 1, Bi$_1$ = 1, l/R_1 = 6 at different Peclet numbers.

The influence of convective cooling of the disk face (the Biot number) on the maximum temperature $T^{**}_{\max}$ is shown in Fig. 4b.

A coefficient of thermal energy distribution η (19) versus the widths of the zone of contact for different Pe numbers is plotted in Fig. 5a. At a fixed width of the contact zone, an increase in the speed of disk rotation increases the amount of friction heat transferred to the disk.

The corresponding results for several Bi numbers are depected in Fig. 5b. With enhancement of convective cooling for a fixed source width of $2\theta_0$, the amount of heat consumed for heating of the working surface of the disk increases, though its temperature, as seen in Fig. 4b, decreases.

Figure 6 shows η as a function of the dimensionless disk half-width Δ.

Conclusions. A mathematical model is suggested for calculating the heat distribution coefficient over a sliding zone of contact of a rapidly rotating disk and a fixed round rod. Such a friction pair often serves as a working element of experimental friction machines. Quasistationary and stationary heat conduction problems are solved for the disk and the rod, respectively. From the condition of equality of the mean temperatures in the zone of contact an analytical expression for the heat distribution coefficient is obtained. The influence of the speed of disk rotation, convective cooling of its surface, and geometric dimensions on the heat distribution between the contacting bodies is investigated.

NOTATION

T, temperature; q, q_1, heat fluxes transferred to disk and rod, respectively; k, thermal diffusivity; K, thermal conductivity; h, h', coefficients of heat release from disk face and side surface, respectively; h_1, heat transfer coefficient from side surface of rod; R, disk radius; R_1, radius of rod cross-section; δ, half-thickness of disk; l, rod length; A, actual area of contact zone; ω, angular speed of disk rotation; r, θ, polar coordinates; θ_0, angular half-width of heating zone on disk surface; $I_0(\cdot)$, $I_1(\cdot)$, modified Bessel functions of the first kind.

REFERENCES

1. H. Carslaw and D. Jaeger, Conduction of Heat in Solids, Clarendon Press, Oxford (1959).
2. I. N. Sneddon, The Use of Integral Transforms, New York (1972).
3. B. Jecim and V. Winner, Problemy Treniya, 106, No. 1, 93-100 (1984).
4. M. Abramovits and I. Stigan, Handbook on Special Functions [in Russian], Moscow (1979).